14. A perpetuity paying 1 at the beginning of each 6-month period has a present value of 20 .

A second perpetuity pays X at the beginning of every 2 years.

Assuming the same annual effective interest rate, the two present values are equal.

Determine X .

(A) 3.5

(B) 3.6

(C) 3.7

(D) 3.8

(E) 3.9

39. Sally lends 10,000 to Tim. Tim agrees to pay back the loan over 5 years with monthly payments payable at the end of each month.

Sally can reinvest the monthly payments from Tim in a savings account paying interest at 6%, compounded monthly. The yield rate earned on Sally's investment over the five-year period turned out to be 7.45%, compounded semi-annually.

What nominal rate of interest, compounded monthly, did Sally charge Tim on the loan?

(A) 8.53%

(B) 8.59%

(C) 8.68%

(D) 8.80%

(E) 9.16%

47. Jim began saving money for his retirement by making monthly deposits of 200 into a fund earning 6% interest compounded monthly. The first deposit occurred on January 1, 1985 .

Jim became unemployed and missed making deposits 60 through 72 . He then continued making monthly deposits of 200 .

How much did Jim accumulate in his fund on December 31, 1999 ?

(A) 53,572

(B) 53,715

(C) 53,840

(D) 53,966

(E) 54,184

9. Victor invests 300 into a bank account at the beginning of each year for 20 years. The account pays out interest at the end of every year at an annual effective interest rate of $i\%$. The interest is reinvested at an annual effective rate of $\left(\frac{i}{2}\right)\%$.

The yield rate on the entire investment over the 20 year period is 8% annual effective.

Determine i.

(A) 9%

(B) 10%

(C) 11%

(D) 12%

(E) 13%

20. Sandy purchases a perpetuity-immediate that makes annual payments. The first payment is 100, and each payment thereafter increases by 10 .

Danny purchases a perpetuity-due which makes annual payments of 180 .

Using the same annual effective interest rate, $i > 0$, the present value of both perpetuities are equal.

Calculate i .

(A) 9.2%

(B) 9.7%

(C) 10.2%

(D) 10.7%

(E) 11.2%

22. Jerry will make deposits of 450 at the end of each quarter for 10 years.

At the end of 15 years, Jerry will use the fund to make annual payments of Y at the beginning of each year for 4 years, after which the fund is exhausted.

The annual effective rate of interest is 7% .

Determine Y .

(A) 9573

(B) 9673

(C) 9773

(D) 9873

(E) 9973

44. Joe can purchase one of two annuities:

Annuity 1: A 10-year decreasing annuity-immediate, with annual payments of $10, 9, 8, \ldots, 1$.

Annuity 2: A perpetuity-immediate with annual payments. The perpetuity pays 1 in year 1, 2 in year 2, 3 in year 3, . . . , and 11 in year 11. After year 11, the payments remain constant at 11.

At an annual effective interest rate of i, the present value of Annuity 2 is twice the present value of Annuity 1.

Calculate the value of Annuity 1.

(A) 36.4

(B) 37.4

(C) 38.4

(D) 39.4

(E) 40.4

5. A perpetuity-immediate pays X per year. Brian receives the first n payments, Colleen receives the next n payments, and Jeff receives the remaining payments. Brian's share of the present value of the original perpetuity is 40%, and Jeff's share is K.

Calculate K.

(A) 24%

(B) 28%

(C) 32%

(D) 36%

(E) 40%

17. At an annual effective interest rate of i, $i > 0\%$, the present value of a perpetuity paying 10 at the end of each 3-year period, with the first payment at the end of year 6, is 32 .

At the same annual effective rate of i, the present value of a perpetuity-immediate paying 1 at the end of each 4-month period is X.

Calculate X.

(A) 38.8

(B) 39.8

(C) 40.8

(D) 41.8

(E) 42.8

26. Susan invests Z at the end of each year for seven years at an annual effective interest rate of 5%. The interest credited at the end of each year is reinvested at an annual effective rate of 6%. The accumulated value at the end of seven years is X.

Lori invests Z at the end of each year for 14 years at an annual effective interest rate of 2.5%. The interest credited at the end of each year is reinvested at an annual effective rate of 3%. The accumulated value at the end of 14 years is Y.

Calculate $\frac{Y}{X}$.

(A) 1.93

(B) 1.98

(C) 2.03

(D) 2.08

(E) 2.13

48. A company's stock is currently selling for 28.50 . Its next dividend, payable one year from now, is expected to be 0.50 per share. Analysts forecast a long-run dividend growth rate of 7.5% for the company.

Tomorrow the long-run dividend growth rate estimate changes to 7% .

Calculate the new stock price.

(A) 22.2

(B) 23.8

(C) 25.9

(D) 28.0

(E) 28.5

50. The present values of the following three annuities are equal:

(i) perpetuity-immediate paying 1 each year, calculated at an annual effective interest rate of 7.25%

(ii) 50-year annuity-immediate paying 1 each year, calculated at an annual effective interest rate of $j\%$

(iii) n-year annuity-immediate paying 1 each year, calculated at an annual effective interest rate of $j - 1\%$

Calculate n.

(A) 30

(B) 33

(C) 36

(D) 39

(E) 42

5. Mike buys a perpetuity-immediate with varying annual payments. During the first 5 years, the payment is constant and equal to 10 . Beginning in year 6, the payments start to increase. For year 6 and all future years, the current year's payment is $K\%$ larger than the previous year's payment.

At an annual effective interest rate of 9.2%, the perpetuity has a present value of 167.50 .

Calculate K, given $K < 9.2$.

(A) 4.0

(B) 4.2

(C) 4.4

(D) 4.6

(E) 4.8

12. To accumulate 8000 at the end of $3n$ years, deposits of 98 are made at the end of each of the first n years and 196 at the end of each of the next $2n$ years.

The annual effective rate of interest is i. You are given $(1 + i)^n = 2.0$.

Determine i.

(A) 11.25%

(B) 11.75%

(C) 12.25%

(D) 12.75%

(E) 13.25%

16. Olga buys a 5-year increasing annuity for X.

Olga will receive 2 at the end of the first month, 4 at the end of the second month, and for each month thereafter the payment increases by 2 .

The nominal interest rate is 9% convertible quarterly.

Calculate X.

(A) 2680

(B) 2730

(C) 2780

(D) 2830

(E) 2880

35. At time $t = 0$, Sebastian invests 2000 in a fund earning 8% convertible quarterly, but payable annually.

He reinvests each interest payment in individual separate funds each earning 9% convertible quarterly, but payable annually.

The interest payments from the separate funds are accumulated in a side fund that guarantees an annual effective rate of 7%.

Determine the total value of all funds at $t = 10$.

(A) 3649

(B) 3964

(C) 4339

(D) 4395

(E) 4485

8. Kathryn deposits 100 into an account at the beginning of each 4-year period for 40 years. The account credits interest at an annual effective interest rate of i.

The accumulated amount in the account at the end of 40 years is X, which is 5 times the accumulated amount in the account at the end of 20 years.

Calculate X.

(A) 4695

(B) 5070

(C) 5445

(D) 5820

(E) 6195

22. A perpetuity costs 77.1 and makes annual payments at the end of the year.

The perpetuity pays 1 at the end of year 2, 2 at the end of year 3,..., n at the end of year $(n+1)$. After year $(n+1)$, the payments remain constant at n. The annual effective interest rate is 10.5%.

Calculate n.

(A) 17

(B) 18

(C) 19

(D) 20

(E) 21

26. 1000 is deposited into Fund X, which earns an annual effective rate of 6%. At the end of each year, the interest earned plus an additional 100 is withdrawn from the fund. At the end of the tenth year, the fund is depleted.

The annual withdrawals of interest and principal are deposited into Fund Y, which earns an annual effective rate of 9%.

Determine the accumulated value of Fund Y at the end of year 10.

(A) 1519

(B) 1819

(C) 2085

(D) 2273

(E) 2431

33. At an annual effective interest rate of i, $i > 0$, both of the following annuities have a present value of X:

(i) a 20-year annuity-immediate with annual payments of 55

(ii) a 30-year annuity-immediate with annual payments that pays 30 per year for the first 10 years, 60 per year for the second 10 years, and 90 per year for the final 10 years

Calculate X.

(A) 575

(B) 585

(C) 595

(D) 605

(E) 615

45. A perpetuity-immediate pays 100 per year. Immediately after the fifth payment, the perpetuity is exchanged for a 25-year annuity-immediate that will pay X at the end of the first year. Each subsequent annual payment will be 8% greater than the preceding payment.

Immediately after the 10^{th} payment of the 25-year annuity, the annuity will be exchanged for a perpetuity-immediate paying Y per year.

The annual effective rate of interest is 8%.

Calculate Y.

(A) 110

(B) 120

(C) 130

(D) 140

(E) 150

1. Which of the following expressions does NOT represent a definition for $a_{\overline{n}|}$?

(A) $v^n\left[\dfrac{(1+i)^n-1}{i}\right]$

(B) $\dfrac{1-v^n}{i}$

(C) $v+v^2+\ldots+v^n$

(D) $v\left[\dfrac{1-v^n}{1-v}\right]$

(E) $\dfrac{s_{\overline{n}|}}{(1+i)^{n-1}}$

4. An estate provides a perpetuity with payments of X at the end of each year. Seth, Susan, and Lori share the perpetuity such that Seth receives the payments of X for the first n years and Susan receives the payments of X for the next m years, after which Lori receives all the remaining payments of X.

Which of the following represents the difference between the present value of Seth's and Susan's payments using a constant rate of interest?

(A) $X\left[a_{\overline{n}|} - v^n a_{\overline{m}|}\right]$

(B) $X\left[\ddot{a}_{\overline{n}|} - v^n \ddot{a}_{\overline{m}|}\right]$

(C) $X\left[a_{\overline{n}|} - v^{n+1} a_{\overline{m}|}\right]$

(D) $X\left[a_{\overline{n}|} - v^{n-1} a_{\overline{m}|}\right]$

(E) $X\left[v\, a_{\overline{n}|} - v^{n+1} a_{\overline{m}|}\right]$

9. The present value of a series of 50 payments starting at 100 at the end of the first year and increasing by 1 each year thereafter is equal to X. The annual effective rate of interest is 9%.

Calculate X.

(A) 1165

(B) 1180

(C) 1195

(D) 1210

(E) 1225

12. Which of the following are characteristics of all perpetuities?

I. The present value is equal to the first payment divided by the annual effective interest rate.

II. Payments continue forever.

III. Each payment is equal to the interest earned on the principal.

(A) I only

(B) II only

(C) III only

(D) I, II, and III

(E) The correct answer is not given by (A), (B), (C), or (D).

14. An annuity-immediate pays 20 per year for 10 years, then decreases by 1 per year for 19 years. At an annual effective interest rate of 6%, the present value is equal to X.

Calculate X.

(A) 200

(B) 205

(C) 210

(D) 215

(E) 220

17. At an annual effective interest rate of i, the present value of a perpetuity-immediate starting with a payment of 200 in the first year and increasing by 50 each year thereafter is 46,530.

Calculate i.

(A) 3.25%

(B) 3.50%

(C) 3.75%

(D) 4.00%

(E) 4.25%

20. An investor wishes to accumulate 10,000 at the end of 10 years by making level deposits at the beginning of each year. The deposits earn a 12% annual effective rate of interest paid at the end of each year. The interest is immediately reinvested at an annual effective interest rate of 8%.

Calculate the level deposit.

(A) 541

(B) 572

(C) 598

(D) 615

(E) 621

21. A discount electronics store advertises the following financing arrangement:

"We don't offer you confusing interest rates. We'll just divide your total cost by 10 and you can pay us that amount each month for a year."

The first payment is due on the date of sale and the remaining eleven payments at monthly intervals thereafter.

Calculate the effective annual interest rate the store's customers are paying on their loans.

(A) 35.1%

(B) 41.3%

(C) 42.0%

(D) 51.2%

(E) 54.9%

23. The stock of Company X sells for 75 per share assuming an annual effective interest rate of i. Annual dividends will be paid at the end of each year forever.

The first dividend is 6, with each subsequent dividend 3% greater than the previous year's dividend.

Calculate i.

(A) 8%

(B) 9%

(C) 10%

(D) 11%

(E) 12%

24. An annuity pays 1 at the end of each year for n years. Using an annual effective interest rate of i, the accumulated value of the annuity at time $(n + 1)$ is 13.776 . It is also known that $(1 + i)^n = 2.476$.

Calculate n .

(A) 4

(B) 5

(C) 6

(D) 7

(E) 8

3. An investor accumulates a fund by making payments at the beginning of each month for 6 years. Her monthly payment is 50 for the first 2 years, 100 for the next 2 years, and 150 for the last 2 years. At the end of the 7th year the fund is worth 10,000.

The annual effective interest rate is i, and the monthly effective interest rate is j.

Which of the following formulas represents the equation of value for this fund accumulation?

(A) $\ddot{s}_{\overline{24}|j}(1+i)\left[(1+i)^4+2(1+i)^2+3\right]=200$

(B) $\ddot{s}_{\overline{24}|j}(1+j)\left[(1+j)^4+2(1+j)^2+3\right]=200$

(C) $\ddot{s}_{\overline{24}|j}(1+i)\left[(1+i)^4+2(1+i)^2+3\right]=200$

(D) $s_{\overline{24}|j}(1+i)\left[(1+i)^4+2(1+i)^2+3\right]=200$

(E) $s_{\overline{24}|j}(1+j)\left[(1+j)^4+2(1+j)^2+3\right]=200$

8. Matthew makes a series of payments at the beginning of each year for 20 years. The first payment is 100. Each subsequent payment through the tenth year increases by 5% from the previous payment. After the tenth payment, each payment decreases by 5% from the previous payment.

Calculate the present value of these payments at the time the first payment is made using an annual effective rate of 7%.

(A) 1375

(B) 1385

(C) 1395

(D) 1405

(E) 1415

9. A company deposits 1000 at the beginning of the first year and 150 at the beginning of each subsequent year into perpetuity.

In return the company receives payments at the end of each year forever. The first payment is 100. Each subsequent payment increases by 5%.

Calculate the company's yield rate for this transaction.

(A) 4.7%

(B) 5.7%

(C) 6.7%

(D) 7.7%

(E) 8.7%

12. Megan purchases a perpetuity-immediate for 3250 with annual payments of 130. At the same price and interest rate, Chris purchases an annuity-immediate with 20 annual payments that begin at amount P and increase by 15 each year thereafter.

Calculate P.

(A) 90

(B) 116

(C) 131

(D) 176

(E) 239

13. For 10,000, Kelly purchases an annuity-immediate that pays 400 quarterly for the next 10 years.

Calculate the annual nominal interest rate convertible monthly earned by Kelly's investment.

(A) 10.0%

(B) 10.3%

(C) 10.5%

(D) 10.7%

(E) 11.0%

14. Payments of X are made at the beginning of each year for 20 years. These payments earn interest at the end of each year at an annual effective rate of 8%. The interest is immediately reinvested at an annual effective rate of 6%. At the end of 20 years, the accumulated value of the 20 payments and the reinvested interest is 5600.

Calculate X.

(A) 121.67

(B) 123.56

(C) 125.72

(D) 127.18

(E) 128.50

20. The dividends of a common stock are expected to be 1 at the end of each of the next 5 years and 2 for each of the following 5 years. The dividends are expected to grow at a fixed rate of 2% per year thereafter.

Assume an annual effective interest rate of 6%.

Calculate the price of this stock using the dividend discount model.

(A) 29

(B) 33

(C) 37

(D) 39

(E) 41

23. The present value of a 25-year annuity-immediate with a first payment of 2500 and decreasing by 100 each year thereafter is X.

Assuming an annual effective interest rate of 10%, calculate X.

(A) 11,346

(B) 13,615

(C) 15,923

(D) 17,396

(E) 18,112

49.

Happy and financially astute parents decide at the birth of their daughter that they will need to provide 50,000 at each of their daughter's 18th, 19th, 20th and 21st birthdays to fund her college education. They plan to contribute X at each of their daughter's 1st through 17th birthdays to fund the four 50,000 withdrawals. They anticipate earning a constant 5% annual effective interest rate on their contributions.

Let $v = 1/1.05$.

Determine which of the following equations of value can be used to calculate X.

(A) $X\sum_{k=1}^{17} v^k = 50,000[v + v^2 + v^3 + v^4]$

(B) $X\sum_{k=1}^{16} 1.05^k = 50,000[1 + v + v^2 + v^3]$

(C) $X\sum_{k=0}^{17} 1.05^k = 50,000[1 + v + v^2 + v^3]$

(D) $X\sum_{k=1}^{17} 1.05^k = 50,000[1 + v + v^2 + v^3]$

(E) $X\sum_{k=0}^{17} v^k = 50,000[v^{18} + v^{19} + v^{20} + v^{21} + v^{22}]$

~~50.~~ Deleted

84.

John made a deposit of 1000 into a fund at the beginning of each year for 20 years.

At the end of 20 years, he began making semiannual withdrawals of 3000 at the beginning of each six months, with a smaller final withdrawal to exhaust the fund. The fund earned an annual effective interest rate of 8.16%.

Calculate the amount of the final withdrawal.

(A) 561

(B) 1226

(C) 1430

(D) 1488

(E) 2240

85.

The present value of a perpetuity paying 1 every two years with first payment due immediately is 7.21 at an annual effective rate of i.

Another perpetuity paying R every three years with the first payment due at the beginning of year two has the same present value at an annual effective rate of $i + 0.01$.

Calculate R.

(A) 1.23

(B) 1.56

(C) 1.60

(D) 1.74

(E) 1.94

86.

A loan of 10,000 is repaid with a payment made at the end of each year for 20 years. The payments are 100, 200, 300, 400, and 500 in years 1 through 5, respectively. In the subsequent 15 years, equal annual payments of X are made.

The annual effective interest rate is 5%.

Calculate X.

(A) 842
(B) 977
(C) 1017
(D) 1029
(E) 1075

87.

An investor wishes to accumulate 5000 in a fund at the end of 15 years. To accomplish this, she plans to make equal deposits of X at the end of each year for the first ten years. The fund earns an annual effective rate of 6% during the first ten years and 5% for the next five years.

Calculate X.

(A) 224
(B) 284
(C) 297
(D) 312
(E) 379

92.

You are given the following term structure of interest rates:

Length of investment in years	Spot rate
1	7.50%
2	8.00%
3	8.50%
4	9.00%
5	9.50%
6	10.00%

Calculate the one-year forward rate for the fifth year implied by this term structure.

(A) 9.5%
(B) 10.0%
(C) 11.5%
(D) 12.0%
(E) 12.5%

93.

Seth has two retirement benefit options.

His first option is to receive a lump sum of 374,500 at retirement.

His second option is to receive monthly payments for 25 years starting one month after retirement. For the first year, the amount of each monthly payment is 2000. For each subsequent year, the monthly payments are 2% more than the monthly payments from the previous year.

Using an annual nominal interest rate of 6%, compounded monthly, the present value of the second option is P.

Determine which of the following is true.

(A) P is 323,440 more than the lump sum option amount.
(B) P is 107,170 more than the lump sum option amount.
(C) The lump sum option amount is equal to P.
(D) The lump sum option amount is 60 more than P.
(E) The lump sum option amount is 64,090 more than P.

96.

An investor's retirement account pays an annual nominal interest rate of 4.2%, convertible monthly.

On January 1 of year y, the investor's account balance was X. The investor then deposited 100 at the end of every quarter. On May 1 of year $(y + 10)$, the account balance was $1.9X$.

Determine which of the following is an equation of value that can be used to solve for X.

(A) $\dfrac{1.9X}{(1.0105)^{\frac{124}{3}}} + \sum_{k=1}^{42} \dfrac{100}{(1.0105)^{k-1}} = X$

(B) $X + \sum_{k=1}^{42} \dfrac{100}{(1.0035)^{3(k-1)}} = \dfrac{1.9X}{(1.0035)^{124}}$

(C) $X + \sum_{k=1}^{41} \dfrac{100}{(1.0035)^{3k}} = \dfrac{1.9X}{(1.0035)^{124}}$

(D) $X + \sum_{k=1}^{41} \dfrac{100}{(1.0105)^{k}} = \dfrac{1.9X}{(1.0105)^{\frac{124}{3}}}$

(E) $X + \sum_{k=1}^{42} \dfrac{100}{(1.0105)^{k-1}} = \dfrac{1.9X}{(1.0105)^{\frac{124}{3}}}$

97.

Five deposits of 100 are made into a fund at two-year intervals with the first deposit at the beginning of the first year.

The fund earns interest at an annual effective rate of 4% during the first six years and at an annual effective rate of 5% thereafter.

Calculate the annual effective yield rate earned over the investment period ending at the end of the tenth year.

(A) 4.18%

(B) 4.40%

(C) 4.50%

(D) 4.58%

(E) 4.78%

98.

John finances his daughter's college education by making deposits into a fund earning interest at an annual effective rate of 8%. For 18 years he deposits X at the beginning of each month.

In the 16^{th} through the 19^{th} years, he makes a withdrawal of 25,000 at the beginning of each year. The final withdrawal reduces the fund balance to zero.

Calculate X.

(A) 207

(B) 223

(C) 240

(D) 245

(E) 260

99.

Jack inherited a perpetuity-due, with annual payments of 15,000. He immediately exchanged the perpetuity for a 25-year annuity-due having the same present value. The annuity-due has annual payments of X.

All the present values are based on an annual effective interest rate of 10% for the first 10 years and 8% thereafter.

Calculate X.

(A) 16,942

(B) 17,384

(C) 17,434

(D) 17,520

(E) 18,989

102.

A woman worked for 30 years before retiring. At the end of the first year of employment she deposited 5000 into an account for her retirement. At the end of each subsequent year of employment, she deposited 3% more than the prior year. The woman made a total of 30 deposits.

She will withdraw 50,000 at the beginning of the first year of retirement and will make annual withdrawals at the beginning of each subsequent year for a total of 30 withdrawals. Each of these subsequent withdrawals will be 3% more than the prior year. The final withdrawal depletes the account.

The account earns a constant annual effective interest rate.

Calculate the account balance after the final deposit and before the first withdrawal.

(A) 760,694

(B) 783,948

(C) 797,837

(D) 805,541

(E) 821,379

103.

An insurance company purchases a perpetuity-due providing a geometric series of quarterly payments for a price of 100,000 based on an annual effective interest rate of i. The first and second quarterly payments are 2000 and 2010, respectively.

Calculate i.

(A) 10.0%

(B) 10.2%

(C) 10.4%

(D) 10.6%

(E) 10.8%

124.

Rhonda purchases a perpetuity providing a payment of 1 at the beginning of each year. The perpetuity's Macaulay duration is 30 years.

Calculate the modified duration of this perpetuity.

(A) 28.97
(B) 29.00
(C) 29.03
(D) 29.07
(E) 29.10

125.

Stocks F and J are valued using the dividend discount model. The required annual effective rate of return is 8.8%. The dividend of Stock F has an annual growth rate of g and the dividend of Stock J has an annual growth rate of $-g$.

The dividends of both stocks are paid annually on the same date.

The value of Stock F is twice the value of Stock J. The next dividend on Stock F is half of the next dividend on Stock J.

Calculate g.

(A) 0.0%
(B) 0.8%
(C) 2.9%
(D) 5.3%
(E) 8.8%